OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4729/01**

MATHEMATICS

Mechanics 2

TUESDAY 16 JANUARY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 A uniform solid cylinder has height 20 cm and diameter 12 cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cylinder topples when the angle of inclination is α. Find α. [3]

2 Two smooth spheres A and B, of equal radius and of masses 0.2 kg and 0.1 kg respectively, are free to move on a smooth horizontal table. A is moving with speed $4\,\text{m s}^{-1}$ when it collides directly with B, which is stationary. The collision is perfectly elastic. Calculate the speed of A after the impact. [4]

3 A small sphere of mass 0.2 kg is projected vertically downwards with speed $21\,\text{m s}^{-1}$ from a point at a height of 40 m above horizontal ground. It hits the ground and rebounds vertically upwards, coming to instantaneous rest at its initial point of projection. Ignoring air resistance, calculate

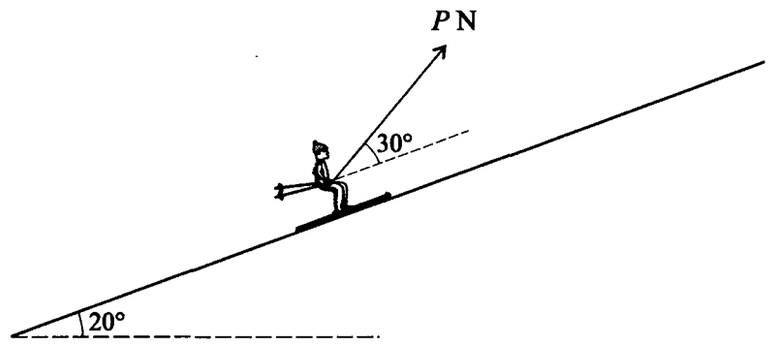
(i) the coefficient of restitution between the sphere and the ground, [6]

(ii) the magnitude of the impulse which the ground exerts on the sphere. [2]

4 A skier of mass 80 kg is pulled up a slope which makes an angle of 20° with the horizontal. The skier is subject to a constant frictional force of magnitude 70 N. The speed of the skier increases from $2\,\text{m s}^{-1}$ at the point A to $5\,\text{m s}^{-1}$ at the point B, and the distance AB is 25 m.

(i) By modelling the skier as a small object, calculate the work done by the pulling force as the skier moves from A to B. [5]

(ii)



It is given that the pulling force has constant magnitude P N, and that it acts at a constant angle of 30° above the slope (see diagram). Calculate P. [3]

5 A model train has mass 100 kg. When the train is moving with speed $v\,\mathrm{m\,s^{-1}}$ the resistance to its motion is $3v^2$ N and the power output of the train is $\frac{3000}{v}$ W.

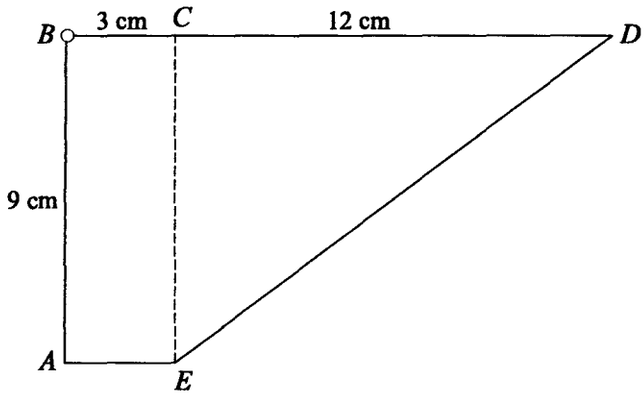
(i) Show that the driving force acting on the train is 120 N at an instant when the train is moving with speed $5\,\mathrm{m\,s^{-1}}$. [2]

(ii) Find the acceleration of the train at an instant when it is moving horizontally with speed $5\,\mathrm{m\,s^{-1}}$. [2]

The train moves with constant speed up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{98}$.

(iii) Calculate the speed of the train. [5]

6



A uniform lamina $ABCDE$ of weight 30 N consists of a rectangle and a right-angled triangle. The dimensions are as shown in the diagram.

(i) Taking x- and y-axes along AE and AB respectively, find the coordinates of the centre of mass of the lamina. [8]

The lamina is freely suspended from a hinge at B.

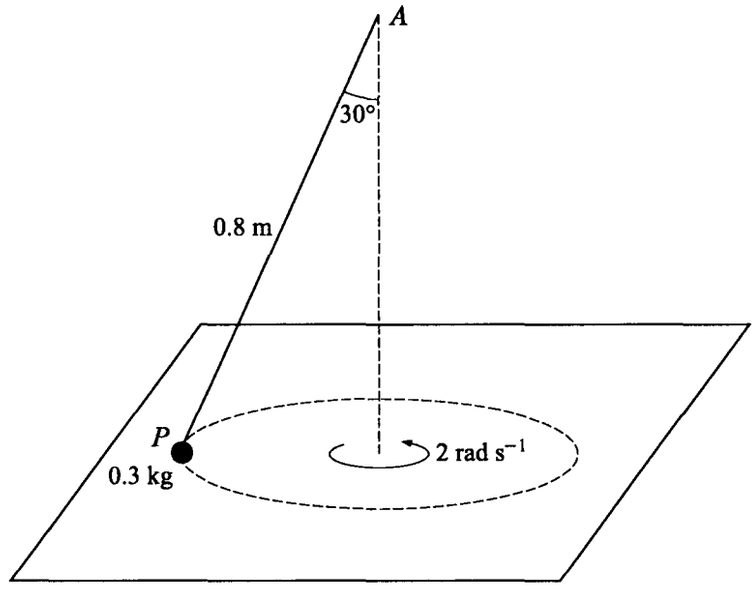
(ii) Calculate the angle that AB makes with the vertical. [2]

The lamina is now held in a position such that BD is horizontal. This is achieved by means of a string attached to D and to a fixed point 15 cm directly above the hinge at B.

(iii) Calculate the tension in the string. [3]

[Questions 7 and 8 are printed overleaf.]

7



One end of a light inextensible string of length 0.8 m is attached to a fixed point A which lies above a smooth horizontal table. The other end of the string is attached to a particle P, of mass 0.3 kg, which moves in a horizontal circle on the table with constant angular speed 2 rad s^{-1}. AP makes an angle of 30° with the vertical (see diagram).

(i) Calculate the tension in the string. [4]

(ii) Calculate the normal contact force between the particle and the table. [3]

The particle now moves with constant speed v m s^{-1} and is on the point of leaving the surface of the table.

(iii) Calculate v. [6]

8 A missile is projected with initial speed 42 m s^{-1} at an angle of 30° above the horizontal. Ignoring air resistance, calculate

(i) the maximum height of the missile above the level of the point of projection, [3]

(ii) the distance of the missile from the point of projection at the instant when it is moving **downwards** at an angle of 10° to the horizontal. [11]

1		com directly above lowest point	B1			
		tan α = 6/10	M1			
		α = 31.0	A1	3	or 0.540 rads	**3**

2		e = 1 = (y–x)/4	B1		or ½x0.2x^2 + ½x0.1y^2 =	
		0.8 = 0.2x + 0.1y	B1		½x0.2x4^2(B1/B1 for any 2)	
		solving sim. equ.	M1		not if poor quad. soln.	
		x = 4/3 only	A1	4		**4**

3	(i)	$x^2 = 21^2$ + 2x40x9.8	M1			
		x = 35	A1			
		0 = y^2 – 2x40x9.8	M1			
		y = 28	A1		may be implied	
		e = 28/35	M1			
		e = 0.8	A1	6	aef	
	(ii)	0.2x28 – – 0.2x35	M1		must be double negative	
		I = 12.6	A1	2		**8**

4	(i)	½x80x5^2 or ½x80x2^2 either KE	B1		1000/160	
		70 x 25	B1		1750	
		80x9.8x25sin20°	B1		6703.6	
		WD=½x80x5^2-½x80x2^2+70x25+80x9.8x25sin20°	M1		4 parts	
		9290	A1	5		
	(ii)	Pcos30°x25	B1		or a=0.42	
		Pcos30°.25=9290 / Pcos30°-70-80x9.8sin20°=80a	M1			
		P = 429 /if P found 1st then Pcos30°x25 =9290 ok	A1	3		**8**

5	(i)	D = 3000/5^2 = 120	M1			
			A1	2	**AG**	
	(ii)	120 – 75 = 100a	M1			
		a = 0.45 ms^{-2}	A1	2		
	(iii)	100x9.8x1/98	B1		weight component	
		3000/v^2=3v^2+100x9.8x1/98	M1			
		3000 = 3v^4 + 10v^2	A1		aef	
		solving quad in v^2	M1		(v^2 = 30)	
		v = 5.48 ms^{-1}	A1	5	accept √30	**9**

6	(i)	com of Δ 4 cm right of C	B1			
		1.5 x 10 + 7 x 20 = $\bar{x}$ x 30	M1			
			A1			
		$\bar{x}$ = 5.17	A1		5 1/6 31/6	
		com of Δ 6 cm above E	B1		or 3 cm below C	
		4.5 x 10 + 6 x 20 = $\bar{y}$ x 30	M1			
			A1			
		$\bar{y}$ = 5.5	A1	8		
	(ii)	tanθ = 5.17/3.5	M1		right way up and (9– $\bar{y}$)	
		55.9° or 124°	A1√	2	√ their $\bar{x}$ /(9 – $\bar{y}$)	
	(iii)	d = 15sin45° (10.61)	B1		dist to line of action of T	
		Td = 30 x 5.17	M1		allow Tx15 i.e. T vertical	
		T = 14.6	A1	3		**13**

7	(i)	$T\sin30°$	B1			
		$T\sin30° = 0.3x0.4x2^2$	M1		resolving horizontally	
			A1			
		$T = 0.96$	A1	4		
	(ii)	$R + T\cos30° = 0.3x9.8$	M1		resolving vertically	
			A1			
		$R = 2.11$	A1√	3	√ their T (2.94—Tcos30°)	
	(iii)	$T_1\sin30° = 0.3 \times v^2/0.4$	M1		or $0.3x0.4x\omega^2$	
			A1		$(T_1 = 1.5v^2)$	
		$T_1\cos30° = 0.3 \times 9.8$	B1		$(T_1 = 1.96\sqrt{3} = 3.3948)$	
		$R = 0$	B1		may be implied or stated	
		$\tan30°=v^2/(0.4 \times 9.8)$for elim of T_1	M1		and $v=0.4\omega$ ($\omega = 3.76$)	
		$v = 1.50$	A1	6		**13**

8	(i)	$v_v = 42\sin30°$ (=21)	B1			
		$0 = 21^2 – 2x9.8xh$	M1			
		$h = 22.5$	A1	3		
	(ii)	$v_h = 42\cos30°$ (=36.4)	B1			
		$v_v = \pm v_h \times \tan10°$	M1			
		$v_v = \pm6.41$ or $21\sqrt{3}\tan10°$	A1		or $42\cos30°.\tan10°$	
		$–6.41 = 42\sin30° – 9.8t$	M1	**	must be –6.41(also see "or" x 2)	
		$t = 2.80$	A1	**		
		$y=42\sin30°x2.8 – 4.9x2.8^2$	M1	**		
		$y = 20.4$	A1√	**	√ their t	
		$x = 42\cos30° \times 2.80$	M1			
		$x = 102$	A1√		√ their t	
		$\sqrt{(x^2 + y^2)}$	M1			
		$d = 104$	A1	11		
	or	$6.41^2 = 21^2 + 2 \times -9.8s$	M1	**	vert dist first then time	
		s = 20.4	A1	**		
		$20.4 = 21t + ½. -9.8t^2$	M1	**		
		$t = 2.80$	A1	**		
	or	22.5 — s and $6.41^2=2x9.8s$	M1	**	dist from top (s = 2.096)	
		y = 20.4	A1	**		
		22.5 & $2.1 = ½.9.8t^2$	M1	**	2 separate times (2.143, 0.654)	
		$t = 2.80$	A1	**	2.143 + 0.654	**14**
		alternatively				
	(ii)	$y = x/\sqrt{3} – x^2/270$ aef	B1		$y=x\tan30°– 9.8x^2/2.42^2.\cos^230°$	
		$dy/dx = 1/\sqrt{3} – x/135$	M1		for differentiating	
			A1		aef	
		$dy/dx = – \tan10°$	M1		must be $–\tan10°$	
		$1/\sqrt{3} – x/135 = – \tan10°$	A1			
		solve for x	M1			
		$x = 102$	A1√		√ on their dy/dx	
		$y = x/\sqrt{3} – x^2/270$	M1			
		$y = 20.4$	A1√		√ their x	
		$\sqrt{(x^2 + y^2)}$	M1			
		$d = 104$	A1	(11)		